MAKING SENSE OF THE MULTIPLE MEANINGS OF 'EMBODIED MATHEMATICS LEARNING'

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The theme of 'embodiment' has become an important approach in current mathematics education research, growing in significance from the mid-1990s onward. However terminology of 'body' and 'embodiment' is used to signal multiple, widely varying meanings in this research. Studies are grounded in a number of radically different theoretical bases, with the result that mathematics education researchers do not necessarily mean the same thing at all when they refer to 'body', 'embodiment' and 'embodied mathematics learning'. In this theoretical paper, the author offers a framework for interpreting these polysemous terms in relation to their theoretical groundings, with examples from the mathematics education literature.

MULTIPLE MEANINGS OF 'EMBODIMENT' AND THEIR SOURCES

In this conceptual paper, I offer a snapshot of the current research in embodied mathematics education. I develop an account of the historical context, disciplinary origins, research aims and meanings accruing to 'body' in the various strands that make up this research, and then undertake a brief annotated bibliography that attempts to characterize a selection of studies in these terms. Many papers in this area will be grounded in several foundational strands, but I posit that there will be one strand that predominates. This sense-making process may serve as a guide to researchers working in embodiment research in our field and offer a first attempt at a conceptual framework not previously available in this area.

HISTORICAL BACKGROUND: EMBODIMENT AS A SIGNIFICANT RESEARCH STRAND IN MATHEMATICS EDUCATION

In recent years, many mathematics educators have begun to challenge Platonic, Cartesian and Bourbakian assumptions that positioned mathematics (and mathematics teaching and learning) as wholly abstract, mental and disembodied (Roth 2010; Radford 2002; and many others). Such assumptions, based on the premise of a human mind-body split and of the transcendence of mind over body, were predominant in Western philosophy from the time of Plato (circa 500 BCE) till the mid-20th century. Since that time, there has been philosophical opposition to the postulate of mind as separate from, and superior to body. Philosophical challenges to the mind-body split have accelerated since the early 1980s, and mathematics education research has taken this up as a significant basis for research since the mid-1990s.

In many ways, mathematics and mathematics education offer an important space for the consideration of embodiment and conceptualization, since mathematics has been considered the *sine qua non* of abstract idealization since Plato's time. For example, in

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a famous passage in Plato's *Meno* (1976), Socrates teaches an unschooled slave boy about what we would identify as irrational numbers, ostensibly by activating a memory of a realm of perfect, disembodied mathematical Forms via Socratic questioning. From Ancient Greece to the Bourbaki school of mathematicians in the mid-20th century, who famously banned geometric sketches of triangles as excessively embodied (Yaglom 1981), mathematics has been a prime exemplar of non-bodily ways of knowing.

For this reason, mathematics has also been an important area for bringing embodiment back into theories of cognition, learning and representation; if mathematical knowing of abstract concepts can be convincingly shown to involve 'body', then so can almost any other realm of human knowledge.

Within mathematics education research, the degree of inclusion (or exclusion) of 'body' in theories of mathematical knowing has the potential to affect theories of mathematics learning, pedagogy, learning materials, curriculum, classrooms and learning spaces, assessment, teacher-student relationships and many other facets of the teaching and learning of mathematics. A fundamental paradigm shift from the assumption of disembodied to embodied ways of knowing in research can change almost everything about the theory and practices of mathematics education, and for this reason, the turn towards 'body' is an important one.

EMBODIMENT AS A CONTEMPORARY RESEARCH THEME ACROSS DISCIPLINES

Since the mid-20th century, theorists in all disciplines have begun to reconsider and reframe concepts of 'body' and 'embodiment', and to move away from Cartesian mind-body dualism. This conceptual shift has accelerated since 1980, and has affected nearly every field of intellectual endeavour and praxis (for example, see Canning 1999). We might well ask why this is happening in our era – why our cultural preoccupation with embodiment at this time? Questions about the reasons that particular intellectual trends, schools of thought or new ideas arise at a particular time and place are seldom resolved to everyone's satisfaction, even in retrospect, since so many convergences (political, economic, academic, technological, social, religious, etc.) might account for them. I have written elsewhere (Gerofsky forthcoming) one way of understanding 'why embodiment now?', based in McLuhan's theoretical approaches to culture and technology (McLuhan & McLuhan 1988), but it is beyond the scope of this paper to discuss this here.

Universes of discourse around embodiment vary widely. As researchers in mathematics education, this poses some dilemmas. Our field has traditions of borrowing, adapting, transforming and re-envisioning theories drawn from widely heterogeneous origins. Mathematics educators often re-make these theories in surprising and generative new ways in adapting them for new purposes. In this process, conflicting meanings may arise from shared terminology. I will examine this polysemy here, looking at theoretical groundings, research aims and conceptualizations of 'body'

that collocate in a number of prototypical research approaches to embodied mathematics learning.

ASPECTS OF EMBODIED RESEARCH STRANDS: THEORETICAL GROUNDING, RESEARCH AIMS, CONCEPTUALIZATION OF 'BODY'

Embodiment research in mathematics education to date has been grounded in the following *theoretical domains*: philosophy, semiotics, cultural studies, linguistics/cognitive linguistics, computer science, cognitive neuroscience, education/ curriculum and pedagogy, gesture studies and fine and performing arts.

To clarify these terms: philosophy refers here primarily to Western traditions of classical to Modernist philosophical thought, but may also include philosophical traditions outside the Western canon. Semiotics arose in the 20th century to analyze cultural phenomena via a consideration of signs and their signification, and is closely connected with structuralism, with roots in linguistics mathematics, philosophy, anthropology, and literary criticism. Cultural studies is the postmodern domain of theory that situates knowledge in the particularities of bodies, cultures, places, genders, classes, 'races', ethnicities, ages, abilities, etc., often focusing on the relationships between particular ways of knowing based in embodied experiences and the ways these knowledges play out in power structures like colonialism and political struggles.

Linguistic studies language, and cognitive linguistics focuses on the relationship between language and the human mind and conceptualization. Computer science includes theories and research in HCI (human-computer interactions) and the 'cognitively ergonomic' design of more and less bodily engaged, multisensory interactions between learners and applications. Cognitive neuroscience focuses on the brain, neurological systems and other biological systems as substrates for learning. Curriculum and pedagogy in education focus on understanding and improving teaching and learning, inside and outside of schools. Gesture studies is a new field concentrating on the use of hands and other parts of the body for primarily communicative purposes. Fine and performing arts interact with mathematics/ math education as media for expression of mathematical relationships via performances (theatre, dance, storytelling, music, film) and art objects (sculpture, painting, drawing, textile arts).

Research aims of embodiment studies in mathematics education include the intention to create theory, to understand how people learn, to design better tools and systems that support learning, to design better pedagogy, and to create art.

Studies *conceptualize 'body'* in the following ways: body as source of embodied metaphors; as diverse, culturally mediated artefact; as individual and/or collective human bodies; as part of an ecosystem in the actual world; as adjunct to virtual worlds; as an autonomic system of brain, neurological system and 'peripherals'; as a physical body comprising core, limbs and head; as part of the physical world of performance; as

source of evidence of unconscious processes; as a resource for conscious pedagogy; and as something that should be suppressed or expressed.

The three lists above (theory, aims, concepts of 'body') are clearly somewhat arbitrary and neither exhaustive nor mutually exclusive. They are based in the extant work in embodied mathematics education, and reflect the range of approaches researchers have taken to this point.

Elements of the three lists tend to collocate to form a number of strands of embodiment research in mathematics education, described in the following section. It is important to note that particular studies and papers very often combine several of these strands, as researchers strive to bring together heterogeneous sources of work on embodiment in ways that inform mathematics learning research.

HOW THEORY, AIMS AND CONCEPT OF BODY COLLOCATE IN RESEARCH

The chart (Table 1) below brings together elements from the lists above in an effort to characterize predominant contemporary approaches to embodied learning in mathematics. It is followed by a brief section characterizing a sampling of research papers in terms of the strands identified here.

Theoretical grounding	**Research aims**	**Bodies as…**
Philosophy, often including phenomenology	To create theory	Body as something that should be expressed.
Semiotics	To create theory; to understand how people learn	Individual or collective bodies interacting as part of an ecosystem in the actual world
Cultural theory	To create theory	Bodies as diverse, culturally-mediated artefacts
Linguistics/ cognitive linguistics	To create theory	Bodies as sources of embodied metaphors
Computer science/ cognitive science	To design better tools and systems that support learning	Body as adjunct to virtual worlds. Bodies interacting as part of an ecosystem in the actual world. Bodies as sources of embodied metaphors.
Cognitive neuroscience	To understand how people learn	Individual bodies as brains, neurological systems and 'peripherals'. Bodies as sources of evidence of unconscious cognitive processes.
Curriculum and pedagogy	To create theory; to understand how people learn; to design better pedagogy	Individual & collective bodies interacting as part of an ecosystem in the actual world. Bodies as core, limbs, head, available as a resource for conscious pedagogy. Bodies as something that can be expressed or suppressed.
Gesture studies	To create theory; to understand how people	Individual or collective bodies as core, limbs, head as a source of evidence of

	learn and communicate	unconscious processes. Bodies as something that should be expressed.
Fine and performing arts	To create (mathematical) art	Individual and collective bodies as part of the material world of performance and art-making. Bodies as something that should be expressed.

Table 1

A sample of influential studies can then be characterized by combinations of these research strands:

- Philosophy (Campbell & Dawson 1995; Roth 2010; Roth & Thom 2009 (incorporating pedagogy))
- Semiotics (Presmeg 2006; Radford 2002; Radford 2009 (incorporating gesture theory); Radford, Bardini, Sabena, Diallo, & Simbagoye, 2005; Radford, Edwards & Arzarello 2009; Steinbring 2006 (all incorporating pedagogy))
- Cultural theory (De Freitas 2008; De Freitas & Sinclair 2012 (also incorporating gesture studies and pedagogy); Lave 1997; Mowat & Davis 2010, and Sinclair, De Freitas & Ferrera 2013 (incorporating pedagogy))
- Cognitive linguistics (Edwards 2009 (incorporating gesture studies); Lakoff & Núñez 2000; Nemirovsky & Ferrera 2012 (incorporating pedagogy); Núñez, Edwards & Matos 1999 (incorporating pedagogy))
- Computer science/ cognitive science (Abrahamson 2009; Abrahamson & Trninic 2011; Howison, Trninic, Reinholz & Abrahamson 2011; Jackiw & Sinclair 2009; Kaput, Noss & Hoyles 2002; Sinclair & Gol Tabaghi 2010; Winn 2003 (all incorporating pedagogy))
- Cognitive neuroscience (Campbell 2010)
- Curriculum and pedagogy (Arzarello, Robutti & Bazzini 2005; Drijvers et al 2010 (incorporating computer science); Goldin & Kaput 1996; Nemirovsky et al 2004; Noble, Nemirovsky, Wright & Tierney 2001; Sriraman & English 2005; Stevens & Hall 1998; Tall 2004, 2006)
- Gesture studies (Alibali & Nathan 2012; Cook, Mitchell & Goldin-Meadow 2008; Gerofsky 2010; Hostetter & Alibali 2008; Roth 2001 (incorporating philosophy and pedagogy))
- Fine and performing arts (Gadanidis & Borba 2008; Gadanidis, Hoogland & Sedig 2003; Healy & Sinclair 2007; (all incorporating pedagogy))

CONCLUDING REMARKS

This initial work to characterize and exemplify distinctions in embodied mathematics education research identifies a number of strands based on theoretical grounding, research aims and conceptualization of 'body'. While necessarily provisional, imperfect and incomplete, it is hoped that this schema will offer a useful way for researchers to make sense of this heterogeneous, polysemous new area of research.

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